# Chattering in Dynamic Mathematical Two-Phase Flow Models

Javier Bonilla, Luis José Yebra, Eduardo Zarza and Sebastián Dormido

*Abstract*— **This paper describes a chattering phenomena present in the thermodynamic simulations of steam-water two-phase flow mathematical models. The models were developed with the Modelica modelling language and the Dymola tool. Modelica is an object oriented, acausal modelling language with a library of thermal fluid components, Modelica.Media, which includes an implementation of the IAPWS-IF97 (the International Association for the Properties of Water and Stream - Industrial Formulation 1997). The chattering phenomena is presented in a practical case: *once-trough boiler*, and several approaches are proposed to avoid this practical simulation problem.**

## I. INTRODUCTION

Modelling and simulation of dynamic systems is a powerful tool with a great number of advantages in industrial process design and operation [1]. Models are design prototypes, which allow experimentation with the system. They are essential for the design, testing and validation of automatic control systems and they provide a realistic environment for training system operators. Despite all these advantages, modelling and simulation of dynamic systems is not commonly used in industrial process design and operation of solar power plants. Exceptionally, models have been used for automatic control systems.

In this work, the real system is the CIEMAT-PSA (Centro de Investigaciones Energéticas Medioambientales y Tecnológicas - Plataforma Solar de Almería, a Spanish government research and test center) DISS (DIrect Solar Steam) test facility, a parabolic-trough solar power plant. The heat transfer fluid in the DISS test facility is steam and water in two-phase flow. This technology is known as DSG (Direct Steam Generation) as a separate steam generator is unnecessary to produce steam. (See Fig. 1 for an illustration of the parabolic-trough principle.)

A dynamic object-oriented model was developed to study the DISS test facility behaviour [2]. This paper focuses on the heat transfer fluid model.

Modelica was used as the modelling language because it enables acausal modelling and object-oriented programming. Moreover, Modelica has a thermodynamic properties computation library, called Modelica Media 2.2.1 [3], which follows IAPWS (the International Association for the Properties of Water and Steam) recommendations in its recent IF97 formulation, (Industrial Formulation 1997) [4]. This formulation is optimized for short computing times and low CPU load.

Discontinuities in some thermodynamic properties of the steam-water heat transfer fluid cause chattering, which involves high-frequency oscillation in the numeric integration of dynamic model differential equation systems. Therefore, their simulation takes longer, and in some cases never finishes. Chattering, although it has rarely been studied, is a well-known problem [5], and studies about slice-motion in automatic control systems [6]–[10] are worth mentioning. We have attempted to study the problem at its source instead of avoiding the symptoms. This paper discusses the discontinuities in some thermodynamic properties of the water-steam, the chattering problem, and an approach to solve it.

## II. PARABOLIC-TROUGH SOLAR POWER PLANT. DISS TEST FACILITY

One of the several different solar thermal concentrating technologies available is the parabolic-trough technology. Parabolic-Trough Collectors (PTCs) are solar concentrators which convert the direct solar radiation into thermal energy, heating the heat transfer fluid up to around 675 K. Their high working temperature makes PTCs suitable for supplying heat to industrial processes, replacing traditional fossil fuels [11] [12].

A PTC is basically a mirror in the form of a parabola, which collects solar radiation and concentrates it on the absorber tube located in the parabola's focal line through which the heat transfer fluid is pumped, acquiring thermal energy from the solar radiation, Fig. 1 [13].

The heat transfer fluid used in the DISS test facility is the two-phase-flow steam-water, which circulates in three different states, subcooled liquid, steam-water mixture and superheated steam. This technology, known as DSG, increases overall system efficiency while reducing investment costs, by eliminating the oil previously used as a heat-transfer medium between the solar field and the power block, and thereby also eliminating the need for a heat exchanger.

The DISS test facility was set up and put into operation in 1998 for experimenting with direct generation of high-pressure, high-temperature steam in parabolic-trough collector absorber tubes. The DISS loop is the only facility in the world where two-phase-flow steam-water processes in parabolic-trough collectors can be studied under real solar conditions. It is very appropriate not only for the study and development of control schemes, but also for the study and optimization of the operating procedures that must be implemented in direct steam generation solar fields.

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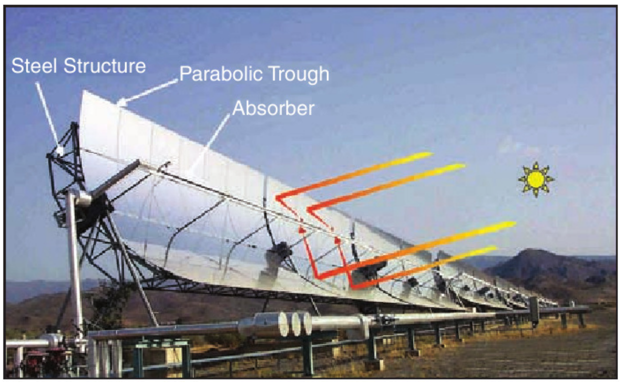


Fig. 1. Main components and working principle of a PTC [13]

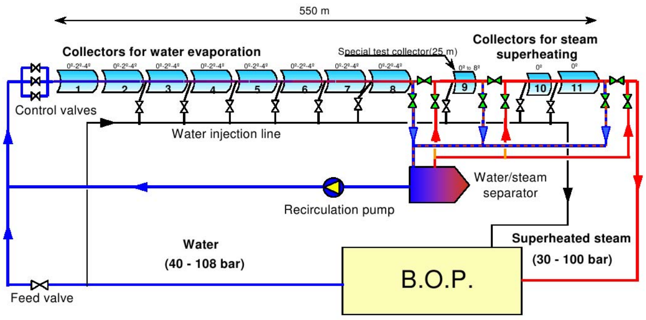


Fig. 2. Diagram of the DISS loop

The DISS loop consists of a solar field where the feedwater is preheated, evaporated and converted into superheated steam as it is circulated through the absorber tubes of a 550-m-long row of parabolic-trough collectors having a total solar collecting surface of 2750 $m^2$. The facility can produce 0.8 kg/s of 10 MPa 675 K steam. Fig. 2 shows a simplified diagram of the DISS loop in which the solar field is represented by one north-south-oriented row of 11 parabolic-trough collectors with rotating axes.

## III. TWO-PHASE FLOW EQUATIONS. IAPWS-IF97 FORMULATION

For developing a computer model of a two-phase-flow steam-water heat transfer fluid, a formulation is necessary. The International Association for the Properties of Water and Steam (IAPWS) in its recently published Industrial Formulation 1997 (IF97), recommends an optimized formulation with short computing times and low CPU load (IAPWS-IF97) [4], consists of a set of regional equations depending on the flow state.

Fig. 3 [4] shows the five regions into which the entire range of IAPWS-IF97 validity is divided with the basic (rectangular boxes) and backward equations, which were developed in such a way that they are numerically very consistent with the corresponding basic equations.

The thermodynamic regions are:

- *Region 1.* Subcooled water.
- *Region 2.* Superheated steam, pressure p $>$ 10 MPa.
- *Region 3.* Water saturation and two-phase flow.

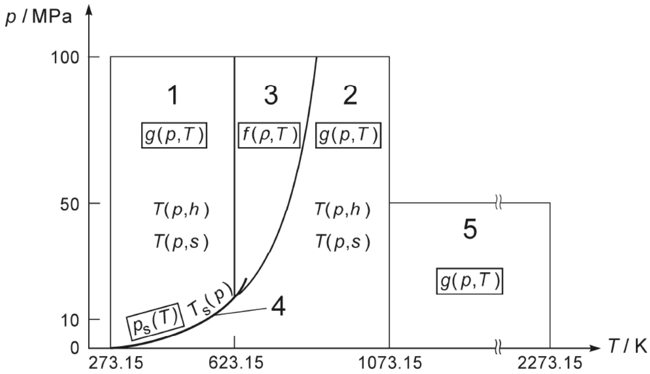


Fig. 3. Structure and regions of IAPWS-IF97 [4]

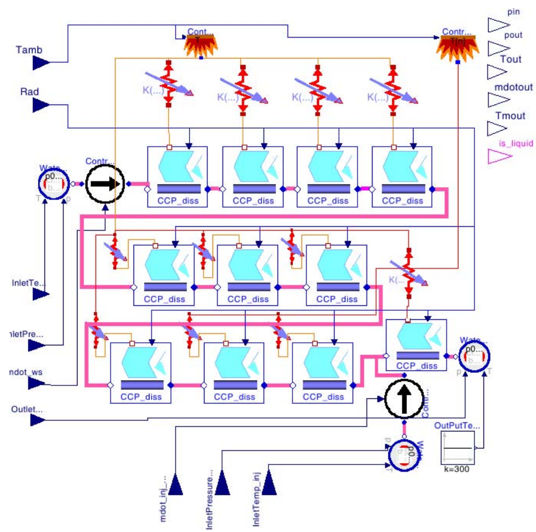


Fig. 4. Modelica model of the DISS test facility [2]

- *Region 4.* Boiling and condensation.
- *Region 5.* Superheated steam, pressure p $<$ 10 MPa and temperature T $\in$[1073.5,2273.5] K.

## IV. OBJECT-ORIENTED MODELING WITH MODELICA

Modelica is a standard unified modelling language [14] with many advantages for modelling dynamic systems because it is both an object-oriented and acausal language. Dynamic behaviour and numerical aspects are taken into account in Modelica, because it provides equations sections and events modelling.

Moreover, Modelica has a library, Modelica.Media 2.2.1, for modelling thermal fluid systems. The previous version of the library was a part of ThermoFluid [15], based mainly on contributions from the Ph.D. theses of Tummescheit [16], Eborn [17] and Wagner [18]. Modelica.Media, and specifically, the Water.IF97_Utilities.BaseIF97 package, which follows the IAPWS-IF97 standard, makes it possible to work with two-phase flow steam-water models.

Fig. 4 shows a components diagram of the DISS test facility Modelica model developed in [2].

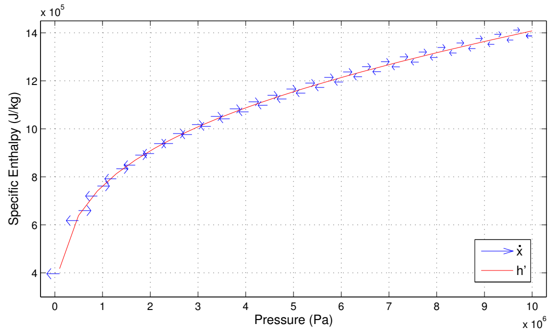


Fig. 5. Velocity field $\dot{x} = (\dot{p}, \dot{h})$ which causes chattering in the boiling curve $h'$

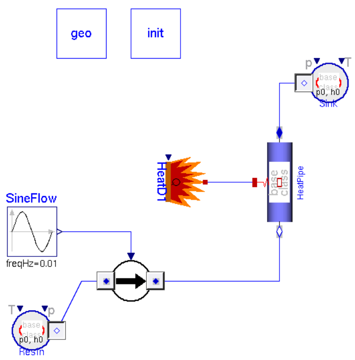


Fig. 6. DiscreteCharacteristic model from ThermoFluid library [15]

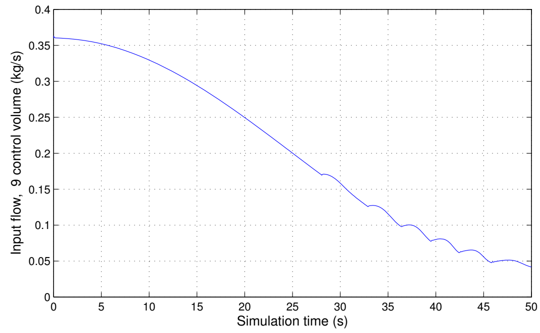


Fig. 7. Bursts of increased flow downstream in the 9th control volume

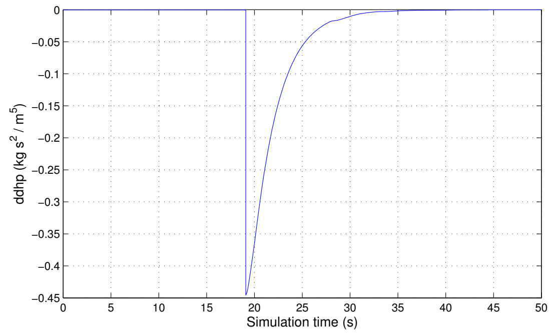


Fig. 8. $\frac{\partial \rho}{\partial h}|_p$ discontinuity, 9th control volume

## V. CHATTERING PROBLEM

### *A. Chattering in the DISS Modelica model*

In the two-phase flow steam-water heat transfer fluid submodel, from the DISS Modelica model [2], under certain circumstances appears a confrontation between the velocity state vectors in the boiling ($h'$) (see Fig. 5) and condensation curves ($h''$). this above-mentioned confrontation causes chattering, that is high-frequency oscilations of the state variables, pressure ($p$) and specific enthalphy ($h$), during simulation. This high-frequency osicilations are due the medium derivatives discontinuities which will be discussed in the following subsections.

### *B. Medium derivatives discontinuities*

A simple model, BoilerPipe.DiscreteCharacteristic, taken from the ThermoFluid library, is presented in this section as an example to focus attention on the medium derivatives discontinuities. The Modelica component diagram is shown in Fig. 6. It has a sinusoidal input water flow which is pumped through a heated pipe discretised into 10 control volumes. As the flow decreases, the boiling barrier moves from the end of the pipe toward the beginning. A plot of the flow inside the pipe during a 50-second simulation shows bursts of increased flow downstream (Fig. 7) as each of the 10 discretised volumes start boiling. This is not good physical behaviour, and can result in high-frequency oscillation in varied two-phase flow simulation caused by discontinuities in the medium derivatives: $\frac{\partial \rho}{\partial h}|_p$ (ddhp, Fig. 8) and $\frac{\partial \rho}{\partial p}|_h$, as the model medium changes from liquid to two-phase flow.

### *C. Source of the chattering in the DISS Modelica model*

Chattering appears when the heat transfer fluid changes state from single-phase to two-phase flow or vice versa in the boiling (the subcooled liquid turns into steam-water mixture, Fig. 9) and condensation curves (the steam-water mixture turns into superheated steam), it is due to discontinuities in thermodynamic properties $\frac{\partial \rho}{\partial h}|_p$ and $\frac{\partial \rho}{\partial p}|_h$ in such curves. Fig. 10 and Fig. 11 show the discontinuities in $\frac{\partial \rho}{\partial h}|_p$ in the boiling curve in two and three dimensions respectively, as a function of the state variables, pressure $p$ and specific enthalpy $h$. Similar discontinuities appear in $\frac{\partial \rho}{\partial p}|_h$ in the boiling curve and in the condensation of both these thermodynamic properties.

### *D. Hypotheses on the chattering problem*

The discontinuities in the thermodynamic properties, $\frac{\partial \rho}{\partial h}|_p$ and $\frac{\partial \rho}{\partial p}|_h$, have been presented. The source of these discontinuities might be any of three possibilities:

- The mathematical models of $\frac{\partial \rho}{\partial h}|_p$ and $\frac{\partial \rho}{\partial p}|_h$ [16] might have discontinuities: This is not true. $\frac{\partial \rho}{\partial h}|_p$ and $\frac{\partial \rho}{\partial p}|_h$ mathematical models are are continuous and differentiable in each region of IAPWS-IF97 and continuous accross the liquid phase boundary, this is proven in [2].
- The Modelica implementation of the IAPWS-IF97 might be incorrect: The Modelica implementation is correct. The source code is available and the same

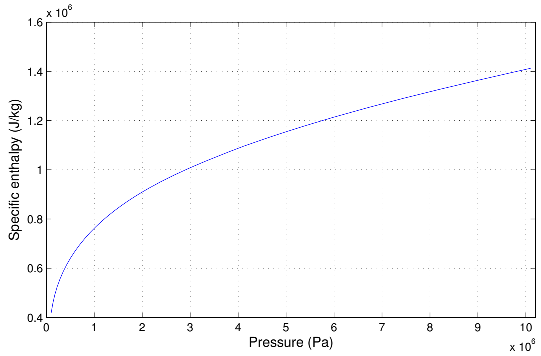


Fig. 9. Boiling curve

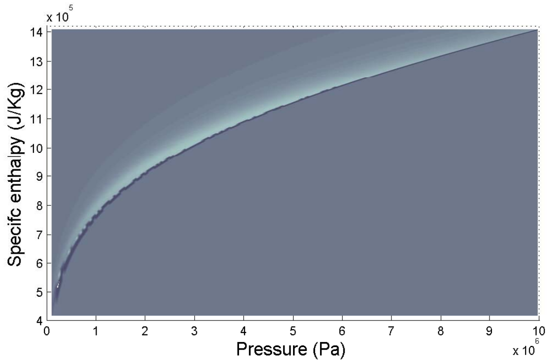


Fig. 10. $\frac{\partial\rho}{\partial h}|_p$ discontinuities in the boiling curve in two dimensions

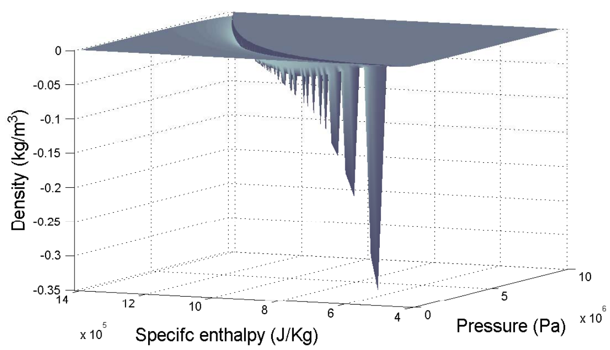


Fig. 11. $\frac{\partial\rho}{\partial h}|_p$ discontinuities in the boiling curve in three dimensions

discontinuities appear in other libraries from different programming languages, like C or Fortran.

- The IAPWS-IF97 might have discontinuities: It is possible. The IAPWS-IF97 equations which lead to the optimized implementation of the mathematical models, may not be completely continuous and differentiable.

The thermodynamic properties in the IAPWS-IF97 are calculated from density, so the problem must be in the density model. Fig. 12 shows density discontinuity during simulation. A two or three dimensional figure is not shown because the discontinuities are smaller than the magnitude of the density and the discontinuities cannot be clearly seen in it.

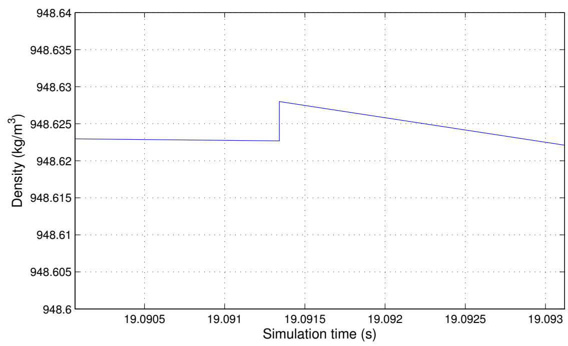


Fig. 12. Density $\rho$ discontinuity, 9th control volume

## VI. APPROACH TO THE CHATTERING PROBLEM

The general solution is to develop continuous and differentiable models for the thermodynamic properties which have discontinuities, to fulfill the numerical integration conditions of mathematical models. The chattering problem is caused by discontinuities in $\frac{\partial\rho}{\partial h}|_p$ and $\frac{\partial\rho}{\partial p}|_h$. These discontinuities are due to discontinuities in the IAPWS-IF97 density model. In the IAPWS-IF97, $\frac{\partial\rho}{\partial h}|_p$ and $\frac{\partial\rho}{\partial p}|_h$ depend on density, so the first step is to develop a continuous and differentiable density model, and after that, to develop continuous and differentiable $\frac{\partial\rho}{\partial h}|_p$ and $\frac{\partial\rho}{\partial p}|_h$ models. These models must depend on density to avoid any kind of inconsistency

The two-phase discontinuity values the are assumed to be correct, because they are based on the perfect mixture hypothesis [19], whereas the single-phase values were found by experimentation.

The goal is to achieve continuous and differentiable models, so some kind of interpolation method has to be used. Bicubic spline interpolation methods were chosen for several reasons. A two-dimensional interpolation method is required because the thermodynamic properties are a function of the state variables, pressure $p$ and specific enthalpy $h$. Bicubic spline interpolation methods allow a smooth surface, and therefore a continuous and differentiable. The slopes at each corner of the surface can be specified using such an interpolation method [20].

An approach have been tried using Hermite bicubic spline interpolation method [21] by density interpolation to achieve a density model which is a function of the whole range of pressure $p$ and specific enthalpy $h$ needed.

### *A. Whole density interpolation*

An interpolated surface is the union of different patches. Four corner points, x-slope, y-slope and twist vector for each corner point are needed to construct a patch. By estimating these values (sample values), continuous differentiable patches and the union between them can be developed to make a surface which must also be continuous and differentiable.

Fig. 13 shows different patches, each patch is the union of four points (corner points), forming a continuous differentiable surface which avoids the discontinuities and mantains a

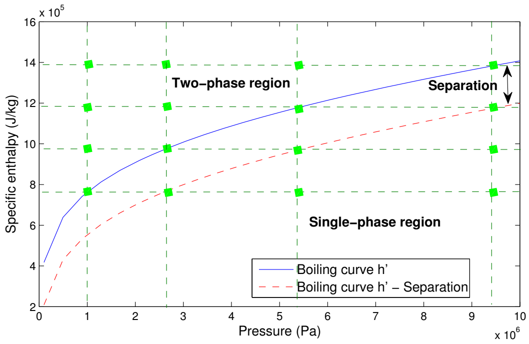


Fig. 13. Grid sampling in the boiling curve for the bicubic spline interpolation method

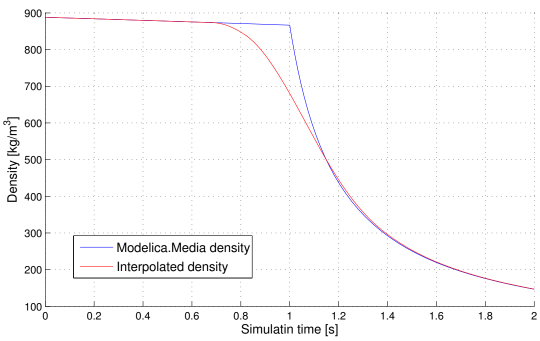


Fig. 14. Comparison between Modelica.Media and interpolated density at constant pressure

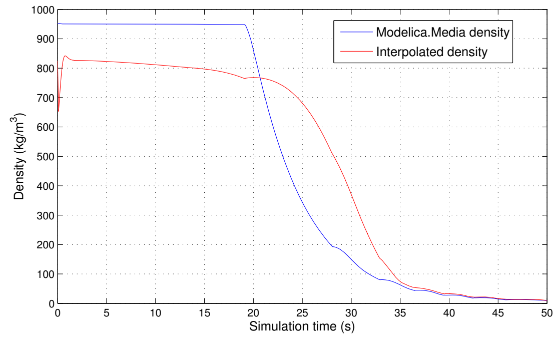


Fig. 15. Comparison between Modelica.Media and interpolated density. 9th control volume

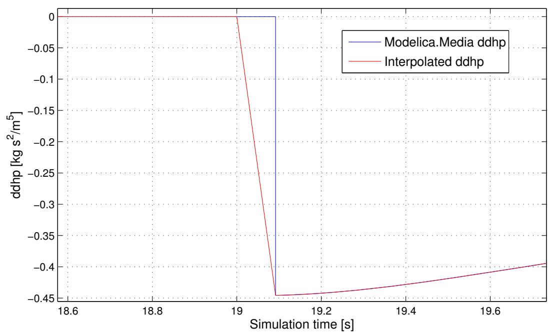


Fig. 16. Comparison between $\frac{\partial\rho}{\partial h}|_p$ using Modelica.Media and interpolated density. 9th control volume

certain distance in the single-phase region. In this approach, the whole thermodynamic property model must be sampled.

It is important to note that the discontinuous area has to be avoided, as shown in Fig. 13, so the sample points cannot be in the area between the boiling curve $h'$ and a certain distance in the single-phase region.

### B. Interpolation results

The density model was therefore sampled using the grid in Fig. 13 but using a narrower separation in pressure and enthalpy, 500 KPa separation in the pressure range $(0, 10]$ MPa and the specific enthalpy values are those in the boiling curve for each pressure, these point are evaluated in the rest of pressure values and those in the separation stripe are rejected. Simulation results are discussed for constant and variable pressure in VI-B.1 and VI-B.2 sections respectively.

*1) Interpolation at constant pressure:* A simple model is presented in this section as an example to study de behaviour of the developed interpolation method at constant pressure. It is just a two-seconds simulation at 15 KPa from 610 KJ/kg as initial specific enthalphy to 915 KJ/kg. The liquid phase boundary is crossed in 1 simulation second.

The results are shown in Fig 14, the interpolated density model is continuous and differentiable for all the different simulation tests carried out. The maxium error is $2.2\%$ with regards the Modelica.Media density values, this error is out of the boiling curve, because in that the interpolated model is smoother and of course the difference whith Modelica.Media density values is larger. Using this approach it is also obtained a continuous (but not differentiable) partial medium derivatives, it will be discussed in the following section.

*2) Interpolation at variable pressure:* The simulation results for variable pressure, shown in Fig 15, are not as good as expected, but they are hopeful. There are no discontinuities in the interpolated density model, but there are no differentiable points either. The results are hopeful because the $\frac{\partial\rho}{\partial h}|_p$ and $\frac{\partial\rho}{\partial p}|_h$ models using interpolated density, instead of the Modelic.Media density, are continuous (although there are no differentiable points) which means that the discontinuities in the medium derivatives are due to the incorrect modelling of density. In Fig. 16 the comparasion beteween $\frac{\partial\rho}{\partial h}|_p$ using Modelica.Media and interpolated density is shown. The interpolated $\frac{\partial\rho}{\partial h}|_p$ is continuous though there is a steep slope in the boiling point.

The no differentiable points in the interpolated density model can be due to patch distribution over the boiling curve. Fig. 17 and the Modelica.Media density model in Fig. 18 show density as a function of the state variables, pressure $p$ and specific enthalpy $h$. A possible solution may be to place the patches along the boiling curve, but in this case, over the whole density model. Moreover, the interpolated density model is rather inaccurate, because the patches do not capture the system dynamics. This can probably be solved by using the same solution, placing the patches along the

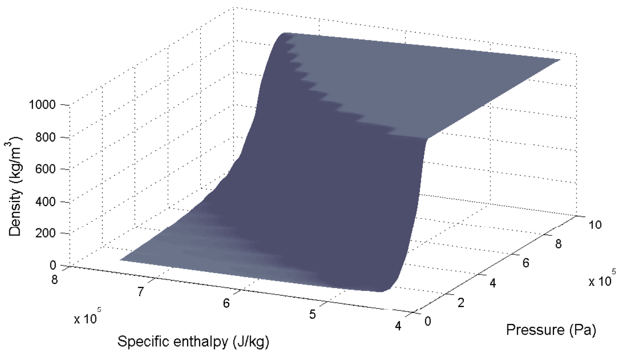


Fig. 17. Interpolated density by Hermite bicubic spline method and grid sampling

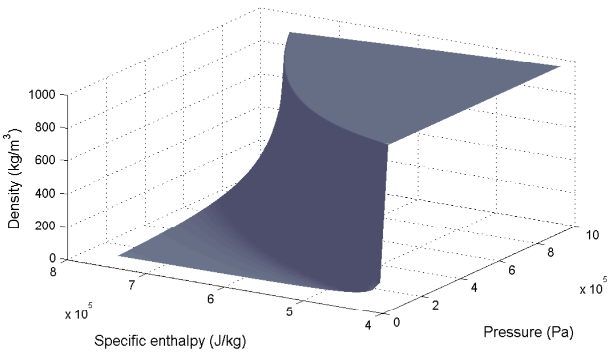


Fig. 18. Modelica.Media density model

boiling curve.

## VII. CONCLUSIONS AND FUTURE WORK

### A. Conclusions

The chattering problem in dynamic mathematical two-phase flow models has been presented. This problem is due to the discontinuities in the thermodynamic properties of water and steam in the IAPWS-IF97, when the heat transfer fluid changes state from single-phase flow to two-phase flow or vice versa in the boiling and condensation curves. The source of these discontinuities is incorrect modelling of the density, with discontinuities, in the IAWPS-IF97 in the same regions.

This paper discussed an attempt at solving this problem at the source instead of avoiding it when it happens. A technique using Hermite bicubic spline interpolation method obtaining continuous but not differentiable density and medium-derivative models have been tested.

### B. Future Work

The main goal is still to achieve a differentiable density model. To do this, different interpolation and the Hermite bicubic spline method must be studied in greater depth.

Once the abovementioned goal has been achieved, the next step is differentiable models for the partial derivatives of density, which have density-model dependent discontinuities. And finally, try to improve the model accuracy and optimize their computing time and CPU load.

## VIII. ACKNOWLEDGMENTS

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